## Post graduate Exam (Basic Engineering Sciences)

Branch: Engineering Mathematics (Master 500)

| Menofia University | Subject: basic topic in <br> Linear Algebra |
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| Faculty of Engineering |  |
| Academic Year: 2015-2016 |  |
| Department: Basic Eng. Sci. | Time Allowed: 3 hours <br> Date: 6/6/2016 <br> Max Marks: 100 |
| Allowed Tables and Charts : None |  |

Answer all the following questions:
Q. 1
(A) Write the tensors terms contained in $S=a_{i j} x^{i} x^{j}$ taking $\boldsymbol{n}=\mathbf{3}$.
(B) Compare between symmetric and anti-symmetric tensors.
(C) Use the mathematical induction to prove that:
i) $(a+b)^{n}=\sum_{k=0}^{n}\binom{n}{k} a^{n-k} b^{k}$
ii) $10^{n+1}+3\left(10^{n}\right)+5 \quad$ is divisible by 9 .
for all natural numbers $\boldsymbol{n}$.
(D) Define the orthogonal and positive definite tensors.
(A) Find a solution of the following linear system , if it is possible:
$x_{1}+2 x_{2}+x_{3}-x_{4}+2 x_{5}=2$
$x_{1}+4 x_{2}+5 x_{3}-3 x_{4}+8 x_{5}=-2$
$-2 x_{1}-x_{2}+4 x_{3}-x_{4}+5 x_{5}=-10$
$3 x_{1}+7 x_{2}+5 x_{3}-4 x_{4}+9 x_{5}=4$
(B) Given $A=\left[\begin{array}{lcc}-4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2\end{array}\right]$
i- Determine the inverse of $A$.
ii- Diagonalize A
iii- Find $A^{n}$
(A) Using Boolean algebra techniques, simplify the expression:

$$
A B+A(B+C)+B(B+C)
$$

(B) Determine the truth table for the following standard POS expression:

$$
(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)
$$

(C) Map the following standard SOP expressions on a Karnaugh map:
i) $\overline{A B} C D+\bar{A} B \bar{C} \bar{D}+A B \bar{C} D+A B C D+A B \bar{C} \bar{D}+\overline{A B} \bar{C} D+A \bar{B} C \bar{D}$
ii) $(\bar{A}+\bar{B}+C+D)+(\bar{A}+B+\bar{C}+\bar{D})+(A+B+\bar{C}+D)$

$$
+(\bar{A}+\bar{B}+\bar{C}+\bar{D})+(A+B+\bar{C}+\bar{D})
$$

(D) Reduce the combinational logic circuit in the following figure to a minimum form.

[Q. 3 (40 mark)]

## With my best wishes

Dr. Z.M. Hendawy

